Int J Theor Phys (2010) 49: 1961–1968
DOI 10.1007/s10773-010-0381-7

The AdS_5 Black Hole Space-time with the Perturbed Dilaton Field Background

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Received: 28 January 2010 / Accepted: 11 May 2010 / Published online: 26 May 2010

Abstract The AdS_5 black hole space-time with perturbed dilaton field background considered. We use the holographic AdS/QCD soft-wall model to investigate the wave functions and the effective potential. In this model, glueballs are described by a massless scalar field in an AdS_5 black hole with a dilaton soft-wall background. For the first time we use modified dilaton field and discuss the consequence of the perturbation. In order to obtain the effective potential we rewrite the equation of motion in the Schrödinger like equation, then try to find corresponding solution.

Keywords AdS/QCD · Black holes · Holography

1 Introduction

In the last decades the AdS/CFT correspondence has been used as an important mathematical tool to investigate the strongly coupled gauge theories [1–6]. The AdS/CFT correspondence relates a ten-dimensional string theory or eleven-dimensional M-theory to a conformal gauge theory on the corresponding space-time boundary. The first example of the AdS/CFT correspondence is the relation between the type IIB string theory in $AdS_5 \times S^5$ space and $\mathcal{N} = 4$ super Yang-Mills gauge theory on the 4-dimensional boundary of AdS_5 space.

The AdS/CFT correspondence may be an important mathematical tool to study many problems in strong interactions (QCD), like the calculation of hadronic spectra. At low energy limit the coupling is strong and perturbation theory is not work. In that case the lattice QCD method or phenomenological effective models are useful. But at high energy limit, in presence of asymptotic freedom, the AdS/QCD approach is available.

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Hard-wall model is the simplest AdS/QCD model which introduces a mass gap via a cutoff on the bulk geometry. In this way it is possible to use the gauge/gravity duality to estimate hadronic masses [7–11]. Another simple model of AdS/QCD, which used in this paper, known as soft-wall, introduces a mass gap through a non-uniform background scalar field in the AdS space-time. The soft-wall model presents linear Regge trajectories for vector mesons and glueballs [12–14].

The calculation of the quasi-normal modes of the massless scalar field in the black hole performed by using the retarded Green's function. It is known that the imaginary part of the Green's function is related to the quasi-normal modes. In order to more study about quasi-normal modes at various backgrounds see Refs. [15–33]. In Ref. [33] the scalar glueballs spectrum at the finite temperature plasma has been studied by using AdS/QCD correspondence and also the quasi-normal modes of the massless scalar field in the AdS black hole and scalar glueballs in a holographic AdS/QCD model at the finite temperature has been studied. The glueballs are described by the massless scalar field in the black hole. According to the Maldacena dictionary spectrum of scalar glueballs on the boundary of space-time are corresponding to the quasi-normal modes of the massless scalar fields in the black hole.

The light scalar and vector glueballs spectra in a holographic QCD with a perturbed dilaton field background originally studied in Ref. [13], also AdS/QCD duality in a perturbed AdS-dilaton background was considered [14]. In the mentioned Refs. the equation of motion for the bulk-to-boundary propagator and AdS two-point correlation function obtained and concluded that the AdS dynamics not affected by dilaton perturbations. Now we would like to apply perturbed dilaton background to the Schrödinger like equation and obtain effect of the perturbation on the effective potential and solutions of the wave function.

This paper organized as the following, in Sect. 2 we give brief review of the soft-wall model. Then in Sect. 3 we recall results of glueballs spectrum in a perturbed AdS-dilaton background. In Sect. 4 we obtain the modified Schrödinger like equation and try to solve it. Finally in Sect. 5 we discuss the effective potential which affected by perturbations. In Sect. 6 we summarized our results and give some suggestions for future works.

2 The Soft-wall Model

In the soft-wall model one deals with a five dimensional AdS space and a background dilaton field $\Phi(z)$. The AdS_5 line element with dilaton field $\Phi(z)$ and geometric function $A(z)$ in the Poincare coordinates is given by,

$$ds^2 = e^{2A(z)}\left[-f(z)\,dt^2 + \sum_{i=1}^{3}(dx^i)^2 + \frac{dz^2}{f(z)}\right], \tag{1}$$

where $f(z) = 1 - \frac{z^4}{z_h^4}$ and z_h is the black hole horizon radius which is related to the black-hole Hawking temperature by the relation $z_h = \frac{1}{\pi T}$. In this configuration $z = 0$ corresponds to the boundary of the space. In presence of background field $\Phi(z)$ in the bulk one can write the action of finite temperature soft-wall model as the following,

$$S = -\frac{\pi^3 L^5}{4\kappa_{10}^2}\int d^5x\sqrt{-g}e^{-\Phi}g^{MN}\partial_M\phi\partial_N\phi, \tag{2}$$

where the factor $e^{-\Phi}\mathcal{L}$ denotes the interaction of the dilaton field with some matter. In the action (2) g_{MN} is the metric given by the relation (1). The parameters L and κ_{10} are the radius of the AdS space and the ten-dimensional gravity constant respectively. Indices M and N run from 0 to 4, so coordinates x^μ ($\mu = 0, 1, 2, 3$) is for four-dimensional boundary and $x^4 = z$ is extra coordinate along the black hole. The dilaton field and the geometric function in the above expressions are given by,

$$\begin{aligned} \Phi(z) &= c^2 z^2, \\ A(z) &= -\ln z, \end{aligned} \tag{3}$$

to recover the Regge behavior. The parameter c sets the scale of the glueball masses.

By using the equation of motion for the scalar field ϕ in the action (2) one can obtain,

$$\frac{e^\Phi}{\sqrt{-g}} \partial_z (\sqrt{-g} e^{-\Phi} g^{zz} \partial_z \phi) + g^{\mu\nu} \partial_\mu \partial_\nu \phi = 0. \tag{4}$$

Now, by using the following Fourier transformation in the relation (4),

$$\phi(z, x) = \int \frac{d^4 k}{(2\pi)^4} e^{ik.x} \bar{\phi}(z, k), \tag{5}$$

where $k_\mu = (-\omega, p_i)$, one can write,

$$e^B f(z) \partial_z (e^{-B} f(z) \partial_z \bar{\phi}) + (\omega^2 - f(z) q^2) \bar{\phi} = 0, \tag{6}$$

where $q^2 = \Sigma_{i-1}^3 q_i^2$ and $B = \Phi - 3A = c^2 z^2 + 3 \ln z$. Now, by introducing new coordinates $\partial_{r_*} = -f(z) \partial_z$ and new variable $\psi = e^{-\frac{B}{2}} \bar{\phi}$, one can rewrite (6) as the following Schrödinger like equation,

$$\partial_{r_*}^2 \psi + \omega^2 \psi = V \psi, \tag{7}$$

where the effective potential defined as,

$$V = \frac{f(z)}{z^2} \left[q^2 z^2 + \frac{15}{4} + \frac{9}{4} \frac{z^4}{z_h^4} + 2c^2 z^2 \left(1 + \frac{z^4}{z_h^4} \right) + c^4 z^4 f(z) \right]. \tag{8}$$

By using these relations one can obtain the scalar and vector glueballs spectrum respectively as the following,

$$\begin{aligned} m_n^2 &= 4c^2 (n + 2) \\ m_n^2 &= 4c^2 (n + 3). \end{aligned} \tag{9}$$

Corresponding to the asymptotical behavior of the equation of motion of the scalar field one can write two solutions which satisfy the Schrödinger like equation,

$$\begin{aligned} \psi_1 &= z^{\frac{5}{2}} [1 + a_1 r + a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots] \\ \psi_2 &= z^{-\frac{3}{2}} [1 + b_1 z + b_2 z^2 + b_3 z^3 + \cdots] + b \psi_1 \ln(c^2 z^2). \end{aligned} \tag{10}$$

Only none-zero coefficients are,

$$\begin{aligned} a_2 &= \frac{q^2+2c^2-\omega^2}{12} \\ a_4 &= \frac{(q^2+2c^2-\omega^2)^2}{384}+\frac{1}{2z_h^4}+\frac{c^4}{32} \\ b_2 &= \frac{\omega^2-q^2-2c^2}{4} \\ b &= \frac{(\omega^2-q^2)(q^2+4c^2-\omega^2)}{32}. \end{aligned} \tag{11}$$

One can check that the effective potential at the $r \to r_h$ vanishes, so the Schrödinger like equation has the other solutions which interpreted as incoming and outgoing wave functions and denoted by ψ_+ and ψ_-. The negative sign interpreted as the incoming plane wave in to the horizon and the positive sign interpreted as the outgoing plane wave from the horizon. Therefore one can write the following expansion as the near horizon solution of the Schrödinger like equation,

$$\psi_{\pm} = e^{\pm i\omega r_*}\left[1+a_{1(\pm)}\left(1-\frac{z}{z_h}\right)+a_{2(\pm)}\left(1-\frac{z}{z_h}\right)^2+\cdots\right], \tag{12}$$

where

$$\begin{aligned} a_{1(\pm)} &= \frac{6+(q^2+4c^2)z_h^2}{4\pm 2i\omega z_h}, \\ a_{2(\pm)} &= \frac{1}{16\pm 4i\omega z_h}\left[\left(22\pm 4i\omega z_h+(q^2+4c^2)z_h^2\right)a_{1(\pm)}-2z_h^2(q^2+8c^2)-9+4c^4z_h^4\right]. \end{aligned} \tag{13}$$

The $\psi_{\pm}$ forms the basis of any other wave functions, also it may be expand $\psi_{\pm}$ in terms of ψ_1 and ψ_2. In the next section we introduce the effect of the perturbed dilaton field background on the glueballs spectra.

3 The Glueballs Spectrum in a Perturbed AdS-dilaton Background

It is known that the dilaton field background and geometric function should be satisfy the Regge behavior. It means that there is the following conditions,

$$\begin{aligned} &\Phi(z)-A(z) \xrightarrow{z\to 0} \ln z \\ &\Phi(z)-A(z) \xrightarrow{z\to \infty} z^2. \end{aligned} \tag{14}$$

The simplest choice consistent with these conditions given by (3). Also there is another choice for the dilaton field background $\Phi(z)$ which obey the conditions (14). One can perturbed dilaton field background as the following [13],

$$\begin{aligned} \Phi(z) &= c^2z^2+\lambda z \\ A(z) &= -\ln z, \end{aligned} \tag{15}$$

where λ is the perturbation parameter which is small dimensionless parameter. The choice (15) does not effectively modify the Regge behavior of the spectrum. It is found that for small values of the parameter λ the first three spectra of the scalar glueball modified as,

$$\begin{aligned} m_0^2 &= 8c^2 + \lambda\frac{3\sqrt{\pi}}{2} \\ m_1^2 &= 12c^2 + \lambda\frac{27\sqrt{\pi}}{16} \\ m_2^2 &= 16c^2 + \lambda\frac{237\sqrt{\pi}}{128}. \end{aligned} \tag{16}$$

Also the first three spectra of the vector glueball modified as,

$$\begin{aligned} m_0^2 &= 12c^2 + \lambda\frac{189\sqrt{\pi}}{128} \\ m_1^2 &= 16c^2 + \lambda\frac{105\sqrt{\pi}}{64} \\ m_2^2 &= 20c^2 + \lambda\frac{14667\sqrt{\pi}}{8192}. \end{aligned} \tag{17}$$

Above results are agree with the relations (9) for $\lambda = 0$. It have shown that the mass splitting between vector and scalar glueballs increases if λ be negative. These spectra obtained in Ref. [13], now in the next section, which is the main part of this paper, we obtain the modified waves functions and the effective potential corresponding to the Schrödinger like equation.

4 The Modified Schrödinger Like Equation

By using perturbed dilaton background field (15) also one can obtain Schrödinger like equation (7) where the effective potential modified as the following,

$$\begin{aligned} V &= \frac{f(z)}{z^2}\left[q^2z^2 + \frac{15}{4} + \frac{9}{4}\frac{z^4}{z_h^4} + 2c^2z^2\left(1 + \frac{z^4}{z_h^4}\right) + c^4z^4 f(z)\right] \\ &\quad + \frac{f(z)}{z^2}\left[c\lambda z\left(\frac{3}{2} + c^2z^2 + \left(2 - \frac{3}{2z}\right)\frac{z^4}{z_h^4} - c^2\frac{z^6}{z_h^6}\right) + \frac{\lambda^2c^2z^2}{4}f(z)\right]. \end{aligned} \tag{18}$$

Also the coefficients of the solutions (10) and (12) modified as the following,

$$\begin{aligned} a_1 &= \frac{6c\lambda}{35} \\ a_2 &= \frac{q^2 + 2c^2 - \omega^2}{12} + \frac{71}{1680}c^2\lambda^2 \\ a_3 &= \frac{c\lambda}{21}\left[c^2 + \frac{c\lambda}{4} + \frac{71}{1120}c^2\lambda^2 + \frac{83}{280}(q^2 + 2c^2 - \omega^2)\right] \end{aligned}$$

$$a_4 = \frac{(q^2+2c^2-\omega^2)^2}{384} + \frac{1}{2z_h^4} + \frac{c^4}{32} - \frac{3c\lambda}{2z_h^4} + c^2\lambda^2\left[\frac{119}{490}c^2 + \frac{c\lambda}{54} + \frac{71}{15680}c^2\lambda^2 + \frac{1097}{13490}(q^2+2c^2-\omega^2)\right] \tag{19}$$

$$b_1 = -\frac{c\lambda}{2}$$

$$b_2 = \frac{\omega^2-q^2-2c^2}{4} + \frac{c^2\lambda^2}{8}$$

$$b_3 = \frac{c^3\lambda^3}{24} - \frac{c\lambda}{6}\left[\omega^2 - q^2 + \frac{3}{4}(\omega^2 - q^2 - 2c^2 + \frac{c^2\lambda^2}{2})\right]$$

$$b = \frac{(\omega^2-q^2)(q^2+4c^2-\omega^2)}{32} + \mathcal{O}(\lambda),$$

and

$$a_{1(\pm)} = \frac{6+(q^2+4c^2)z_h^2}{4\pm 2i\omega z_h},$$

$$a_{2(\pm)} = \frac{1}{16\pm 4i\omega z_h}\left[\left(22\pm 4i\omega z_h + (q^2+4c^2)z_h^2\right)a_{1(\pm)} - 2z_h^2\left(q^2+8c^2-\frac{c^2\lambda^2}{2}\right) - 9 + 4c^4z_h^4\right]. \tag{20}$$

The main consequence of perturbed dilaton background is appearance of the coefficients a_1, a_3, b_1 and b_3. It is clear that the dilaton perturbation have not important effect on the solution of the Schrödinger like equation, so in the first order perturbation $a_{2(\pm)}$ have no change as well as $a_{1(\pm)}$. Also λ dependence of coefficient b is not important here. In the next section we are going to analyze the modified effective potential given by the relation (18).

5 Discussion of the Effective Potential

In this section we would like to discuss about special behaviors of the modified effective potential.

At zero-temperature limit, where $z_h \to \infty$, the effective potential takes the following form,

$$V_{T=0}(z) = \frac{1}{z^2}\left[\frac{15}{4} + (q^2+2c^2)z^2 + c^4z^4c\lambda z\left(\frac{3}{2}+c^2z^2\right) + \frac{\lambda^2c^2z^2}{4}\right]. \tag{21}$$

Also at $c \to 0$ limit one can obtain,

$$V_{c=0}(z) = \frac{f(z)}{z^2}\left(q^2z^2 + \frac{15}{4} + \frac{9}{4}\frac{z^4}{z_h^4}\right), \tag{22}$$

which is corresponding to the case of without dilaton field. Therefore in the above expression the parameter λ is not exist.

By increasing the temperature the effective potential changes. It is known that (for non-perturbed dilaton field background) at high-temperature there are no bound states. This situation is similar to the recent case with perturbed dilaton field background.

At low-temperature limit one can write the effective potential approximately as the following,

$$V(z) = \frac{1}{z^2}\left[\frac{15}{4} - \frac{3}{2}\frac{z^4}{z_h^4} + 2c^2z^2 + c^4z^4 - 2c^4\frac{z^8}{z_h^4}\right] + \frac{1}{z^2}\left[c\lambda\left(\frac{3}{2}z + c^2z^3 - \frac{3}{2}\frac{z^4}{z_h^4} + \frac{1}{2}\frac{z^5}{z_h^4} - 2c^2\frac{z^7}{z_h^4}\right)\right] + \frac{1}{z^2}\left[c^2\lambda^2\left(\frac{z^2}{4} - \frac{z^6}{2z_h^4}\right)\right]. \tag{23}$$

Then by using the relation $z_h = \frac{1}{\pi T}$ one can obtain,

$$V(z) = \frac{1}{z^2}\left[\frac{15}{4} + 2c^2z^2 + c^4z^4 + c\lambda z\left(\frac{3}{2} + c^2z^2\right) + \frac{c^2\lambda^2 z^2}{4}\right] - z^2\pi^4T^4\left[\frac{3}{2} + 2c^4z^4 - c\lambda z\left(\frac{1}{2} - \frac{3}{2z} - 2c^2z^2\right) + \frac{c^2\lambda^2z^2}{2}\right]. \tag{24}$$

The modified effective potential at low-temperature has a constant term as $(2 + \frac{\lambda^2}{4})c^2$, a perturbed oscillator like potential term as $c^4z^2 + c^3\lambda z$, a term with the bare infinity at $z = 0$ as $\frac{15}{4z^2} + \frac{3c\lambda}{2z}$, and finally temperature corrections terms. The expression (24) is agree with the relation (21) at $T = 0$ limit.

6 Conclusion

In this paper we considered the AdS_5-black hole space-time with perturbed dilaton field background. Already the scalar and vector glueballs spectra for perturbed dilaton field background calculated [13]. Here we applied it to the wave equation and obtained the modified wave function and modified effective potential. We have shown that the odd coefficients of the wave functions appear because of perturbed parameter λ. Also we discussed the modified effective potential at zero and finite temperature.

Here, there are interesting problems for future works, for example there are other choices for dilaton field background and geometric function which satisfy the Regge behavior. It is interesting to calculate the wave function and the effective potential for perturbed geometric function.

References

1. Maldacena, J.M.: The large *N* limit of superconformal field theories and supergravity. Adv. Theor. Math. Phys. **2**, 231 (1998)
2. Witten, E.: Anti-de Sitter space and holography. Adv. Theor. Math. Phys. **2**, 253 (1998)
3. Schwart, J.H.: Introduction to *M* Theory and *AdS*/CFT Duality. Lecture Notes in Physics, vol. 525 (1999). hep-th/9812037
4. Petersen, J.L.: Introduction to the Maldacena conjecture on *AdS*/CFT. Int. J. Mod. Phys. A **14**, 3597 (1999)

5. Nastase, H.: Introduction to AdS-CFT. arXiv:0712.0689v2 [hep-th]
6. Klebanov, I.R.: TASI lectures: Introduction to the AdS/CFT correspondence. arXiv:hep-th/0009139
7. Skenderis, K., van Rees, B.C.: Phys. Rev. Lett. **101**, 081601 (2008). arXiv:0805.0150 [hep-th]
8. Skenderis, K., van Rees, B.C.: J. High Energy Phys. **0905**, 085 (2009). arXiv:0812.2909 [hep-th]
9. Berti, E., Cardoso, V., Starinets, A.O.: Class. Quantum Gravity **26**, 163001 (2009). arXiv:0905.2975 [gr-qc]
10. Boschi-Filho, H., Braga, N.R.F.: Eur. Phys. J. C **32**, 529 (2004). arXiv:hep-th/0209080
11. Boschi-Filho, H., Braga, N.R.F.: J. High Energy Phys. **0305**, 009 (2003). arXiv:hep-th/0212207
12. Karch, A., Katz, E., Son, D.T., Stephanov, M.A.: Phys. Rev. D **74**, 015005 (2006). arXiv:hep-ph/0602229
13. Colangelo, P., De Fazio, F., Jugeau, F., Nicotri, S.: Phys. Lett. B **652**, 73 (2007). arXiv:hep-ph/0703316
14. Colangelo, P., De Fazio, F., Giannuzzi, F., Jugeau, F., Nicotri, S.: Phys. Rev. D **78**, 055009 (2008). arXiv:0807.1054 [hep-ph]
15. Berti, E., Cardoso, V., Starinets, A.O.: Class. Quantum Gravity **26**, 163001 (2009). arXiv:0905.2975 [gr-qc]
16. Horowitz, G.T., Hubeny, V.E.: Phys. Rev. D **62**, 024027 (2000). arXiv:hep-th/9909056
17. Berti, E., Cardoso, V., Pani, P.: Phys. Rev. D **79**, 101501 (2009). arXiv:0903.5311 [gr-qc]
18. Kokkotas, K.D., Schmidt, B.G.: Living Rev. Relativ. **2**, 2 (1999). arXiv:gr-qc/9909058
19. Nollert, H.P.: Class. Quantum Gravity **16**, R159 (1999)
20. Wang, B., Molina, C., Abdalla, E.: Phys. Rev. D **63**, 084001 (2001). arXiv:hep-th/0005143
21. Cardoso, V., Lemos, J.P.S.: Phys. Rev. D **63**, 124015 (2001). arXiv:gr-qc/0101052
22. Cardoso, V., Lemos, J.P.S.: Phys. Rev. D **64**, 084017 (2001). arXiv:gr-qc/0105103
23. Starinets, A.O.: Phys. Rev. D **66**, 124013 (2002). arXiv:hep-th/0207133
24. Nunez, A., Starinets, A.O.: Phys. Rev. D **67**, 124013 (2003). arXiv:hep-th/0302026
25. Kovtun, P.K., Starinets, A.O.: Phys. Rev. D **72**, 086009 (2005). arXiv:hep-th/0506184
26. Maeda, K., Natsuume, M., Okamura, T.: Phys. Rev. D **72**, 086012 (2005). arXiv:hep-th/0509079
27. Siopsis, G.: Nucl. Phys. B **715**, 483 (2005). arXiv:hep-th/0407157
28. Miranda, A.S., Zanchin, V.T.: Phys. Rev. D **73**, 064034 (2006). arXiv:gr-qc/0510066
29. Zhang, Y., Jing, J.L.: Int. J. Mod. Phys. D **15**, 905 (2006)
30. Hoyos-Badajoz, C., Landsteiner, K., Montero, S.: J. High Energy Phys. **0704**, 031 (2007). arXiv:hep-th/0612169
31. Miranda, A.S., Zanchin, V.T.: Int. J. Mod. Phys. D **16**, 421 (2007)
32. Amado, I., Hoyos-Badajoz, C.: J. High Energy Phys. **0809**, 118 (2008). arXiv:0807.2337 [hep-th]
33. Miranda, A.S., Ballon Bayona, C.A., Boschi-Filho, H., Braga, N.R.F.: Black-hole quasinormal modes and scalar glueballs in a finite-temperature AdS/QCD model. arXiv:0909.1790 [hep-th]